Supplement Appendix for Measuring the Effects of Fiscal Policy Shocks on U.S. Output in a Markov-Switching Bayesian VAR*

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THIS APPENDIX IS NOT INTENDED FOR PUBLICATION.

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Appendix A.

Data Appendix

The data come from two sources. The fiscal and macro variables are taken from the National Income and Product Accounts (NIPA) tables published by the Bureau of Economic Analysis (BEA). The short- and long-term government interest rates are sourced from the Federal Reserve Bank of St. Louis's FRED database. The quantity variables are converted to real per capita terms by dividing their nominal values by 'Civilian noninstitutional population' (FRED Series ID: CNP16OV) and by the 'Price index for gross domestic product' (NIPA Table 1.1.4, Line 1). The quantity variables are then logged and scaled by 100. Inflation and the interest rates are in percents.

Below are the data sources.

- GOV: 'Government consumption expenditures and gross investment' (NIPA Table 1.1.5, Line 22).
- TAX: 'Government current receipts' (NIPA Table 3.1 Line 1) minus 'Current transfer payments' (NIPA Table 3.1 Line 22) minus 'Government interest payments' (NIPA Table 3.1, Line 27).
- RGDP: 'Gross domestic product' (NIPA Table 1.1.5, Line 1).
- π: Log difference of the 'Price index for gross domestic product' (NIPA Table 1.1.4, Line 1).
- R_{3m}: '3-Month Treasury bill: Secondary market rate' (FRED Series ID: TB3MS).
- R_{10}: '10-Year Treasury Constant Maturity Rate' (FRED Series ID: GS10).

Appendix B.

Verifying Global Identification of the "Non-Recursive Impact Matrix: Extended Blanchard-Perotti" SVAR

This appendix verifies the impact matrix A_0' in "Section 2.4.1: Non-Recursive Impact Matrix: Extended Blanchard-Perotti" satisfies Rubio-Ramírez, Waggoner, and Zha's (2010) necessary and sufficient conditions for global identification. I start by taking the transpose of the impact matrix A_0'. This results in[1]

$$
\begin{array}{ccc}
 & & \begin{array}{cccccc} \text{GOV} & \text{TAX} & \text{AS} & \text{AD} & \text{DF} & \text{TM} \end{array} \\
A_0 = & \begin{array}{c} \text{GOV} \\ \text{TAX} \\ \text{RGDP} \\ \pi \\ \text{R}_{\text{3m}} \\ \text{R}_{\text{10yr}} \end{array} & \begin{bmatrix} a_{11} & 0 & a_{13} & a_{14} & a_{15} & a_{16} \\ 0 & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} \\ 0 & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} \\ 0 & 0 & 0 & a_{44} & a_{45} & a_{46} \\ 0 & 0 & 0 & 0 & a_{55} & a_{56} \\ 0 & 0 & 0 & 0 & 0 & a_{66} \end{bmatrix}, \\
 & q_j & \begin{array}{cccccc} 5 & 4 & 3 & 2 & 1 & 0 \end{array}
\end{array}
$$

where the final row, q_j, counts the number of restrictions on the j-th column for $j = 1, \ldots, 6$. The necessary order condition is satisfied if the number of restrictions is greater than or equal to $n(n-1)/2$, where $n(= 6)$ is the number of endogenous variables. Since the total number of restrictions imposed is $15 = 6(6-1)/2$, the necessary order condition holds.

Next, I construct the restriction matrices Q_j for each $j = 1, \ldots, 6$ equation of the impact

[1] The column labels represent the structural shocks for each behavioral equation. For notational convenience, the column labels are shortened to GOV (Government Spending), TAX (Tax), AS (Aggregate Supply), AD (Aggregate Demand), DF (Debt Financing), and TM (Term Premium).

matrix A_0. The restriction matrices are

$$Q_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \; Q_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$Q_3 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \; Q_4 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$Q_5 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \text{ and } Q_6 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

The corresponding rank matrices are

$$M_1 = \begin{bmatrix} 0 & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} \\ 0 & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} \\ 0 & 0 & 0 & a_{44} & a_{45} & a_{46} \\ 0 & 0 & 0 & 0 & a_{55} & a_{56} \\ 0 & 0 & 0 & 0 & 0 & a_{66} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \ M_2 = \begin{bmatrix} a_{11} & 0 & a_{13} & a_{14} & a_{15} & a_{16} \\ 0 & 0 & 0 & a_{44} & a_{45} & a_{46} \\ 0 & 0 & 0 & 0 & a_{55} & a_{56} \\ 0 & 0 & 0 & 0 & 0 & a_{66} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$M_3 = \begin{bmatrix} 0 & 0 & 0 & a_{44} & a_{45} & a_{46} \\ 0 & 0 & 0 & 0 & a_{55} & a_{56} \\ 0 & 0 & 0 & 0 & 0 & a_{66} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \ M_4 = \begin{bmatrix} 0 & 0 & 0 & 0 & a_{55} & a_{56} \\ 0 & 0 & 0 & 0 & 0 & a_{66} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix},$$

$$M_5 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & a_{66} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \text{ and } M_6 = \begin{bmatrix} 0_{6\times 6} \\ I_6 \end{bmatrix}.$$

Since all of the rank matrices have rank equal to six, Rubio-Ramírez, Waggoner, and Zha's (2010) sufficient rank condition is satisfied. Therefore, the "Non-Recursive Impact Matrix: Extended Blanchard-Perotti" SVAR is globally identified.

Appendix C.

Additional Results

This appendix reports additional results.

Table C1: The Regime Conditional Impact Matrix $\widehat{A}_0'(s_t^{\mathcal{FP}c} = 1 | s_t^{\mathcal{MF}c} = 2)$, MS-BVAR-13, 1960Q1 to 2019Q4

Shock \ Variable	GOV	TAX	RGDP	π	R_{3m}	R_{10yr}
Government Spending	1.219 [0.921, 1.563]					
Tax		0.082 [0.061, 0.103]	-0.486 [-1.157, 0.450]			
Aggregate Supply	-0.468 [-0.675, -0.239]	0.017 [-0.040, 0.054]	1.376 [1.072, 1.777]			
Aggregate Demand	-0.017 [-0.182, 0.143]	0.001 [-0.023, 0.025]	0.176 [-0.089, 0.454]	2.914 [2.195, 3.791]		
Debt Financing	-0.106 [-0.262, 0.082]	-0.001 [-0.019, 0.018]	-0.022 [-0.237, 0.198]	-0.445 [-0.847, -0.083]	2.494 [1.819, 3.443]	
Term Premium	-0.061 [-0.314, 0.160]	-0.008 [-0.053, 0.033]	-0.161 [-0.705, 0.286]	-0.960 [-1.793, -0.159]	-1.468 [-2.296, -0.667]	2.974 [2.106, 5.609]

Notes: The elements of $\widehat{A}_0'(s_t^{\mathcal{FP}c} = 1 | s_t^{\mathcal{MF}c} = 2)$ are at the median of the posterior of MS-BVAR-13. Ninety percent Bayesian credible sets (i.e., 5th and 95th quantiles) are in brackets. The results depend on 10 million MCMC draws. Each row represents a behavioral equation. The behavioral equations are labeled by their respective structural shock. The column labels indicate which variables enter each behavioral equation at impact.

Table C2: The Regime Conditional Impact Matrix $\widehat{A}_0'(s_t^{\mathcal{FP}c} = 2|s_t^{\mathcal{MF}c} = 1)$, MS-BVAR-13, 1960Q1 to 2019Q4

Shock \ Variable	GOV	TAX	RGDP	π	R_{3m}	R_{10yr}
Government Spending	0.908 [0.765, 1.070]					
Tax		0.340 [0.272, 0.406]	-0.675 [-0.935, -0.360]			
Aggregate Supply	-0.174 [-0.345, 0.019]	0.008 [-0.064, 0.057]	0.897 [0.709, 1.106]			
Aggregate Demand	0.044 [-0.080, 0.181]	-0.046 [-0.075, -0.018]	0.024 [-0.155, 0.214]	2.435 [2.068, 2.830]		
Debt Financing	0.130 [0.038, 0.233]	0.001 [-0.019, 0.019]	-0.250 [-0.428, -0.114]	-0.554 [-0.911, -0.267]	0.890 [0.754, 1.035]	
Term Premium	0.055 [-0.120, 0.235]	0.006 [-0.028, 0.039]	-0.102 [-0.348, 0.109]	0.083 [-0.425, 0.643]	-1.077 [-1.399, -0.781]	2.820 [2.241, 3.509]

Notes: See the notes to table C1.

References

Rubio-Ramírez, Juan F., Daniel F. Waggoner, and Tao Zha (2010). "Structural Vector Autoregressions: Theory of Identification and Algorithms for Inference". *Review of Economic Studies* 77.2, pp. 665–696.