Supplement Appendix for Interest Rates, Money, and Fed Monetary Policy in a Markov-Switching Bayesian VAR*

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Appendix A.

An MS-BVAR to Evaluate U.S. Monetary Policy

This appendix describes the methodology used to estimate and conduct inference on the MS-BVARs.

A.1. A Markov-Switching BVAR

Consider the structural MS-BVAR

$$y_t' A_0(s_t) = x_t' A_+(s_t) + \varepsilon_t' \Xi^{-1}(s_t), \quad \varepsilon_t \sim \mathcal{N}(0_{n\times 1}, I_n), \quad t = 1, \ldots, T, \tag{A.1}$$

where s_t denotes an unobservable state (regime) variable and $\Xi(s_t)$ denotes an $n \times n$ diagonal matrix of factor loadings scaling the degree of SV of the structural shocks in ε_t. Gathering all the parameters of each matrix together forms $A_0 \equiv \{A_0(h)\}$, $A_+ \equiv \{A_+(h)\}$, and $\Xi \equiv \{\Xi(h)\}$ for $h = 1, \ldots, H$, where H denotes the total number of states or regimes.

The evolution of s_t follows a Markov process with the transition matrix $Q = [q_{i,j}]$, where $q_{ij} = \text{Prob}\left[s_t = i | s_{t-1} = j\right]$ for $i, j = 1, \ldots, H$. Sims, Waggoner, and Zha (2008) restrict Q to allow only for switching between adjacent regimes. The restricted transition matrix is

$$Q = \begin{bmatrix} q_{11} & (1-q_{22})/2 & \cdots & 0 & 0 \\ 1-q_{11} & q_{22} & \ddots & \vdots & \vdots \\ 0 & (1-q_{22})/2 & \ddots & (1-q_{H-1,H-1})/2 & 0 \\ \vdots & \vdots & \ddots & q_{H-1,H-1} & 1-q_{H,H} \\ 0 & 0 & \cdots & (1-q_{H-1,H-1})/2 & q_{H,H} \end{bmatrix}.$$

The likelihood function of the MS-BVAR (E.1) is

$$p\left(Y_T|\theta, Q\right) = \prod_{t=1}^{T} \left[\sum_{s_t \in H} p\left(y_t | Y_{t-1}, \theta, Q, s_t\right) p\left(s_t | Y_{t-1}, \theta, Q\right) \right], \tag{A.2}$$

where $Y_T \equiv \{y_1, \ldots, y_T\}$, $\theta \equiv \{A_0, A_+, \Xi\}$, and the conditional likelihood function is

$$p\left(y_t|Y_{t-1}, \theta, Q, s_t\right) = (2\pi)^{n/2} \left|\det\left(A_0\left(s_t\right)^{-1'} \Xi\left(s_t\right)^{-1} A_0\left(s_t\right)^{-1}\right)\right|^{-1/2}$$
$$\times \exp\left\{-\frac{1}{2}\left(y_t' A_0\left(s_t\right) - x_t' A_+\left(s_t\right)\right) \Xi\left(s_t\right)^2 \left(y_t' A_0\left(s_t\right) - x_t' A_+\left(s_t\right)\right)\right\}.$$

Evaluating this likelihood requires filtering the sequence of transition probabilities in $p\left(s_t|Y_{t-1}, \theta, Q\right)$.[1]

Given the likelihood function (A.2) and prior, the posterior distribution of θ and Q is

$$p\left(\theta, Q|Y_T\right) \propto p\left(Y_T|\theta, Q\right) p(\theta, Q), \tag{A.3}$$

where the joint prior density for θ and Q is

$$p\left(\theta, Q\right) = \frac{p(\theta)p(Q)}{h} \prod_{t=1}^{T} q_{s_t, s_{t-1}}.$$

A.2. Priors

Posterior distributions of the MS-BVARs are constructed using a prior that has two distinct elements. The first part of the prior relies on Sims and Zha (1998). Their prior is grounded on the belief that each dependent variable in y_t follows an independent random walk process. The behavior of these multivariate random walk processes are governed by six hyperparameters, which are gathered in the vector $\Lambda = [\lambda_0\ \lambda_1\ \lambda_3\ \lambda_4\ \mu_5\ \mu_6]$.[2] Since the MS-BVARs are estimated on quarterly data, $\Lambda = [1.0\ 0.2\ 1.0\ 1.0\ 1.0\ 1.0]$, as suggested by Sims and Zha.

The second part of the prior endows the transition probabilities in Q with a Dirichlet distribution. This prior controls the persistence of each regime. I set the Dirichlet prior to match the average length of a U.S. recession from 1960Q1 to 2018Q4. On this sample, the average NBER dated recession lasts 3.67 quarters, which sets the conditional transition

[1] Appendix A of Sims, Waggoner, and Zha (2008) describes procedures to filter these probabilities.

[2] Sims and Zha (1998) describe these hyperparameters as controlling the overall tightness of the prior on own first lags, λ_0, the relative tightness of the prior on lags of the other $n-1$ variables in y_t, λ_1, the relative tightness of the prior on the rate of lag decay, λ_3, the relative tightness of the prior on the intercept term, λ_4, and the prior beliefs about unit roots, μ_5, and cointegration relationships, μ_6, among the variables in y_t.

probabilities in Q (i.e., q_{ii}) to 0.73.

A.3. Overview of the Estimation and Inference Procedure

The MS-BVARs are estimated and evaluated using the multistage procedure described in Sims, Waggoner, and Zha (2008).[3] This next section summarizes each stage of the estimation and inference procedure; see Sims, Waggoner, and Zha (2008) for more details.

A.3.1. Estimating the Posterior Mode

First, Sims, Waggoner, and Zha (2008) use a blockwise optimization algorithm to estimate the posterior mode of θ and Q in (A.3). Initializing the Metropolis-within-Gibbs MCMC sampler at the posterior mode gives the sampler improved starting values, which increases the efficiency of the sampler and lessens the chance it will become stuck in a local posterior mode.

A.3.2. Drawing from the Posterior using a Metropolis-within-Gibbs MCMC Sampler

Next, the Metropolis-within-Gibbs MCMC sampler of Sims, Waggoner, and Zha (2008) is employed to construct the posterior distributions (A.3) of the MS-BVARs. These posteriors are built using draws from the proposal distribution $p(\theta, Q, S_T|Y_T)$, where $S_T \equiv \{s_1, \dots, s_T\}$. Conditional on the sample data, priors, and $p = 2$,[4] the Metropolis-within-Gibbs sampling algorithm draws from $p(\theta, Q, S_T|Y_T)$ by cycling through the following steps:

Step 1. Initialize $\theta^{(0)}$ and $Q^{(0)}$ at their posterior mode estimates and set $k = 1$.
Step 2. Draw $S_T^{(k)}$ from $p\left(S_T^{(k)}|Y_T, \theta^{(k-1)}, Q^{(k-1)}\right)$.
Step 3. Draw $Q^{(k)}$ from $p\left(Q^{(k)}|Y_T, S_T^{(k)}, \theta^{(k-1)}\right)$.
Step 4. Draw $\theta^{(k)}$ from $p\left(\theta^{(k)}|Y_T, Q^{(k)}, S_T^{(k)}\right)$.

[3] The MS-BVARs are estimated and evaluated using Dynare version 4.5.7 in conjunction with MATLAB R2018b; see Adjemian et al. (2011) for further details. This software runs on the Henry2 Linux cluster at the North Carolina State University High Performance Computing (HPC) Center; see https://www.ncsu.edu/itd/hpc/main.php. Replication codes are available upon request.

[4] The lag length is set to the value that minimizes the Hannan-Quinn Criterion (HQC) of the constant coefficient BVAR. Lag length selection results are available in the online Additional Results appendix.

Step 5. If $k < K_1 + K_2$, set $k = k + 1$ and repeat Steps 2 through 4. Otherwise, stop.

The posterior distribution of each MS-BVAR is constructed using $K_1 + K_2 = 11$ million MCMC draws. The first $K_1 = 1$ million draws are discarded as a burn-in sample to reduce the influence of the initial conditions. This leaves $K_2 = 10$ million draws available for inference.

A.3.3. Model Evaluation

The fit of the estimated constant coefficient and MS-BVARs is evaluated using marginal data densities (MDDs). Gelfand and Dey (1994) and Geweke (2005) develop a modified harmonic mean (HMH) estimator to compute MDDs. This estimator is employed to compute MDDs of the fixed coefficient SVARs. The MS-BVARs are evaluated using MDDs computed using the truncated MHM estimator of Sims, Waggoner, and Zha (2008).

Appendix B.

Necessary and Sufficient Conditions for Global Identification of the Non-Recursive Models

This appendix verifies the non-recursive models satisfy the necessary and sufficient conditions for global identification.[5] Verifying global identification ensures that another parameterization (i.e., model) does not yield the same (or a scalar multiple of the first) likelihood. Rubio-Ramírez, Waggoner, and Zha (2010) provide the tools to check these conditions.

B.1. Verifying Global Identification of the Non-Recursive Identification: Interest Rate Rule Model

Table B1 presents the identifying restrictions for the "Non-Recursive Identification: Interest Rate Rule" model. An X entry in table B1 represents an unrestricted coefficient in the impact matrix A_0. A blank space denotes a zero restriction. Each column represents a behavioral equation. The behavioral equations are labeled at the top by their respective structural shock. The row labels indicate which variables enter each behavioral equation at impact.

The first step in checking for global identification is to verify that the number of restrictions is greater than or equal to $n(n-1)/2$, where $n(=9)$ denotes the number of endogenous variables in the system. This condition is known as Rothenberg (1971)'s order condition.

[5] Rubio-Ramírez, Waggoner, and Zha (2010, pp. 678) show recursive models are always globally identified.

Table B1: Non-Recursive Identifying Restrictions on the Impact Matrix A_0: Interest Rate Rule

Variable \ Shock	Risk Premium	Term Premium	Credit Supply	Credit Demand	Money Demand	Monetary Policy	Aggregate Supply	Aggregate Demand	Labor Supply
	Behavioral Equation								
R_{Baa}	X								
R_{10yr}	X	X							
R_{CP}	X	X	X	X	X				
MI	X	X	X	X					
MB	X	X	X		X				
FFR	X	X	X	X	X	X			
RGDP	X	X	X	X	X	X	X	X	X
P	X	X	X	X	X	X		X	X
UR	X	X	X						X

Notes: Each column in this table represents a behavioral equation. The behavioral equations are labeled at the top by their respective structural shock. The row labels indicate which variables enter each behavioral equation at impact. An X entry denotes an unrestricted impact coefficient, while a blank space denotes a zero restriction.

The order condition is verified by mapping the restrictions from Table B1 to A_0 so that[6]

$$
A_0 = \begin{array}{c} \\ R_{Baa} \\ R_{10yr} \\ R_{CP} \\ MI \\ MB \\ FFR \\ RGDP \\ P \\ UR \\ \\ q_j \end{array}
\begin{array}{c} \begin{array}{ccccccccc} RK & TM & CS & CD & MD & MP & AS & AD & LS \end{array} \\
\begin{bmatrix}
a_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
a_{21} & a_{22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & 0 & 0 & 0 & 0 \\
a_{41} & a_{42} & a_{43} & a_{44} & 0 & 0 & 0 & 0 & 0 \\
a_{51} & a_{52} & a_{53} & 0 & a_{55} & 0 & 0 & 0 & 0 \\
a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} & 0 & 0 & 0 \\
a_{71} & a_{72} & a_{73} & a_{74} & a_{75} & a_{76} & a_{77} & a_{78} & a_{79} \\
a_{81} & a_{82} & a_{83} & a_{84} & a_{85} & a_{86} & 0 & a_{88} & a_{89} \\
a_{91} & a_{92} & a_{93} & 0 & 0 & 0 & 0 & 0 & a_{99}
\end{bmatrix} \\
\begin{array}{ccccccccc} 0 & 1 & 2 & 4 & 4 & 6 & 8 & 7 & 6 \end{array}
\end{array},
$$

where the final row, q_j, denotes the number of restrictions on the j-th column for $j = 1, \dots, 9$. Since the total number of restrictions imposed ($\sum_{j=1}^{9} q_j = 38$) is greater than $n(n-1)/2 = 36$,

[6] For notational convenience, the column labels are shortened to RK (Risk Premium), TM (Term Premium), CS (Credit Supply), CD (Credit Demand), MD (Money Demand), MP (Monetary Policy), AS (Aggregate Supply), AD (Aggregate Demand), and LS (Labor Supply).

Rothenberg (1971)'s order condition holds. However, it is important to note that this order condition is only *necessary but not sufficient* for global identification.

To verify that the model is global identified, Rubio-Ramírez, Waggoner, and Zha (2010)'s sufficient rank condition must also be satisfied. The rank condition investigates whether two parameter points, A_0 and $\tilde{A}_0$, are observationally equivalent (i.e., yield the same likelihood). If so, the model is not globally identified. This condition is satisfied if all the associated rank matrices for each structural equation of the model (which will be defined shortly) have full rank.

Checking the rank condition begins with rearranging the columns of A_0 according to the following ordering rule

$$q_1 \geq q_2 \geq \ldots \geq q_n.$$

Without loss of generality, the updated A_0 matrix is

$$A_0 = \begin{array}{c} \\ \text{RGDP} \\ \text{P} \\ \text{UR} \\ \text{FFR} \\ \text{MB} \\ \text{MI} \\ \text{R}_{\text{CP}} \\ \text{R}_{\text{10yr}} \\ \text{R}_{\text{Baa}} \\ \\ q_j \end{array} \begin{array}{c} \begin{array}{ccccccccc} \text{AS} & \text{AD} & \text{LS} & \text{MP} & \text{MD} & \text{CD} & \text{CS} & \text{TM} & \text{RK} \end{array} \\ \begin{bmatrix} a_{77} & a_{78} & a_{79} & a_{76} & a_{75} & a_{74} & a_{73} & a_{72} & a_{71} \\ 0 & a_{88} & a_{89} & a_{86} & a_{85} & a_{84} & a_{83} & a_{82} & a_{81} \\ 0 & 0 & a_{99} & 0 & 0 & 0 & a_{93} & a_{92} & a_{91} \\ 0 & 0 & 0 & a_{66} & a_{65} & a_{64} & a_{63} & a_{62} & a_{61} \\ 0 & 0 & 0 & 0 & a_{55} & 0 & a_{53} & a_{52} & a_{51} \\ 0 & 0 & 0 & 0 & 0 & a_{44} & a_{43} & a_{42} & a_{41} \\ 0 & 0 & 0 & 0 & a_{35} & a_{34} & a_{33} & a_{32} & a_{31} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{22} & a_{21} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{11} \end{bmatrix} \\ \begin{array}{ccccccccc} 8 & 7 & 6 & 6 & 4 & 4 & 2 & 1 & 0 \end{array} \end{array}.$$

According to Rubio-Ramírez, Waggoner, and Zha (2010), ordering the columns by descending number of column restrictions is the most efficient way to check the rank condition. Although the ordering of the columns is arbitrary, the authors warn that one column ordering might satisfy the rank condition, while another ordering fails to do so. However, the authors show if the above ordering fails to satisfy the rank condition, then all other possible column orderings will also fail. Thus, adhering to the ordering rule established above prevents one from erroneously rejecting a globally-identified model.[7]

The next step is to construct the restriction matrices Q_j for each $j = 1, \ldots, 9$ equation of the updated A_0 matrix. Following Rubio-Ramírez, Waggoner, and Zha (2010), the restriction matrices are

$$Q_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

[7]For example, one can show that the first A_0 matrix's column ordering fails to satisfy the rank condition. The simplest way to verify this claim is to derive the rank matrix associated with the first structural equation. Since there are zero column restrictions in the first column, the corresponding restriction matrix is a 9×9 zero matrix. Multiplying this zero matrix to A_0 and stacking the result on top of $[1\,0\,0\,0\,0\,0\,0\,0\,0]$ forms the first equation's rank matrix. It is straightforward to verify that the first equation's rank matrix has rank equal to one. Therefore, this column ordering leads to the erroneous conclusion that the model is not globally identified. This line of reasoning extends to any identification scheme that imposes zero column restrictions in the first column of the impact matrix; see, for example, the identifications of Robertson and Tallman (2001), Leeper and Zha (2003), and Sims and Zha (2006). One can show after using the established ordering rule that these models are globally identified.

$$Q_2 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, Q_3 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$Q_4 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, Q_5 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$Q_6 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, Q_7 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$Q_8 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, Q_9 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Finally, multiply the Q_j and A_0 matrices and stack the results on top of $\begin{bmatrix} I_j \; 0_{j\times(9-j)} \end{bmatrix}$ for $j = 1, \ldots, 9$. This operation forms the associated rank matrices M_j for each equation of A_0.

The resulting rank matrices are thus given by

$$M_1 = \begin{bmatrix} 0 & a_{88} & a_{89} & a_{86} & a_{85} & a_{84} & a_{83} & a_{82} & a_{81} \\ 0 & 0 & a_{99} & 0 & 0 & 0 & a_{93} & a_{92} & a_{91} \\ 0 & 0 & 0 & a_{66} & a_{65} & a_{64} & a_{63} & a_{62} & a_{61} \\ 0 & 0 & 0 & 0 & a_{55} & 0 & a_{53} & a_{52} & a_{51} \\ 0 & 0 & 0 & 0 & 0 & a_{44} & a_{43} & a_{42} & a_{41} \\ 0 & 0 & 0 & 0 & a_{35} & a_{34} & a_{33} & a_{32} & a_{31} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{22} & a_{21} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{11} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$M_2 = \begin{bmatrix} 0 & 0 & a_{99} & 0 & 0 & 0 & a_{93} & a_{92} & a_{91} \\ 0 & 0 & 0 & a_{66} & a_{65} & a_{64} & a_{63} & a_{62} & a_{61} \\ 0 & 0 & 0 & 0 & a_{55} & 0 & a_{53} & a_{52} & a_{51} \\ 0 & 0 & 0 & 0 & 0 & a_{44} & a_{43} & a_{42} & a_{41} \\ 0 & 0 & 0 & 0 & a_{35} & a_{34} & a_{33} & a_{32} & a_{31} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{22} & a_{21} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{11} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, M_3 = \begin{bmatrix} 0 & 0 & 0 & a_{66} & a_{65} & a_{64} & a_{63} & a_{62} & a_{61} \\ 0 & 0 & 0 & 0 & a_{55} & 0 & a_{53} & a_{52} & a_{51} \\ 0 & 0 & 0 & 0 & 0 & a_{44} & a_{43} & a_{42} & a_{41} \\ 0 & 0 & 0 & 0 & a_{35} & a_{34} & a_{33} & a_{32} & a_{31} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{22} & a_{21} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{11} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$M_4 = \begin{bmatrix} 0 & 0 & a_{99} & 0 & 0 & 0 & a_{93} & a_{92} & a_{91} \\ 0 & 0 & 0 & 0 & a_{55} & 0 & a_{53} & a_{52} & a_{51} \\ 0 & 0 & 0 & 0 & 0 & a_{44} & a_{43} & a_{42} & a_{41} \\ 0 & 0 & 0 & 0 & a_{35} & a_{34} & a_{33} & a_{32} & a_{31} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{22} & a_{21} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{11} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, M_5 = \begin{bmatrix} 0 & 0 & a_{99} & 0 & 0 & 0 & a_{93} & a_{92} & a_{91} \\ 0 & 0 & 0 & 0 & 0 & a_{44} & a_{43} & a_{42} & a_{41} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{22} & a_{21} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{11} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$M_6 = \begin{bmatrix} 0 & 0 & a_{99} & 0 & 0 & 0 & a_{93} & a_{92} & a_{91} \\ 0 & 0 & 0 & 0 & a_{55} & 0 & a_{53} & a_{52} & a_{51} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{22} & a_{21} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{11} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}, M_7 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{22} & a_{21} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{11} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix},$$

$$
M_8 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{11} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}, M_9 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}.
$$

One can easily verify that the ranks of the M_j matrices are equal to nine (i.e., full rank) for $j = 1, \ldots, 9$. Therefore, Rubio-Ramírez, Waggoner, and Zha (2010)'s sufficient rank condition is satisfied.

Since the necessary order and sufficient rank conditions are satisfied, the "Non-Recursive Identification: Interest Rate Rule" model is globally identified.

B.2. Verifying Global Identification of the Non-Recursive Identification: Money Supply Rule Model

The difference between the "Non-Recursive Identification: Money Supply Rule" model (shown in Table B2) and the "Non-Recursive Identification: Interest Rate Rule" model is the location of the zero restriction on MB in the "Credit Demand" column. Specifically, the zero restriction on MB shifts down by one row.

Table B2: Non-Recursive Identifying Restrictions on the Impact Matrix A_0: Money Supply Rule

	Behavioral Equation								
Variable \ Shock	Risk Premium	Term Premium	Credit Supply	Credit Demand	Money Demand	Monetary Policy	Aggregate Supply	Aggregate Demand	Labor Supply
R_{Baa}	X								
R_{10yr}	X	X							
R_{CP}	X	X	X	X	X				
MI	X	X	X	X					
FFR	X	X	X	X	X				
MB	X	X	X		X	X			
RGDP	X	X	X	X	X	X	X	X	X
P	X	X	X	X	X	X		X	X
UR	X	X	X						X

Notes: Each column in this table represents a behavioral equation. The behavioral equations are labeled at the top by their respective structural shock. The row labels indicate which variables enter each behavioral equation at impact. An X entry denotes an unrestricted impact coefficient, while a blank space denotes a zero restriction.

Although this difference seems minor, changing the location of the aforementioned zero restriction creates a new A_0 matrix, i.e.,

$$
A_0 = \begin{array}{c} \\ \text{R}_{\text{Baa}} \\ \text{R}_{\text{10yr}} \\ \text{R}_{\text{CP}} \\ \text{MI} \\ \text{FFR} \\ \text{MB} \\ \text{RGDP} \\ \text{P} \\ \text{UR} \end{array}
\begin{array}{c} \begin{array}{ccccccccc} \text{RK} & \text{TM} & \text{CS} & \text{CD} & \text{MD} & \text{MP} & \text{AS} & \text{AD} & \text{LS} \end{array} \\
\begin{bmatrix}
a_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
a_{21} & a_{22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & 0 & 0 & 0 & 0 \\
a_{41} & a_{42} & a_{43} & a_{44} & 0 & 0 & 0 & 0 & 0 \\
a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & 0 & 0 & 0 & 0 \\
a_{61} & a_{62} & a_{63} & 0 & a_{65} & a_{66} & 0 & 0 & 0 \\
a_{71} & a_{72} & a_{73} & a_{74} & a_{75} & a_{76} & a_{77} & a_{78} & a_{79} \\
a_{81} & a_{82} & a_{83} & a_{84} & a_{85} & a_{86} & 0 & a_{88} & a_{89} \\
a_{91} & a_{92} & a_{93} & 0 & 0 & 0 & 0 & 0 & a_{99}
\end{bmatrix} \end{array}.
$$

$$
q_j \quad 0 \quad 1 \quad 2 \quad 4 \quad 4 \quad 6 \quad 8 \quad 7 \quad 6
$$

The implication is that the global identification results from the previous section cannot be automatically carried over to the "Non-Recursive Identification: Money Supply Rule" model. In this particular case, however, the order condition continues to hold because no additional restrictions are added to the model.

To verify Rubio-Ramírez, Waggoner, and Zha (2010)'s sufficient rank condition, rearrange the columns of A_0 from left to right in descending order so that

$$A_0 = \begin{array}{c} \\ \text{RGDP} \\ \text{P} \\ \text{UR} \\ \text{MB} \\ \text{FFR} \\ \text{MI} \\ \text{R}_{\text{CP}} \\ \text{R}_{\text{10yr}} \\ \text{R}_{\text{Baa}} \\ \\ q_j \end{array} \begin{array}{c} \begin{array}{ccccccccc} \text{AS} & \text{AD} & \text{LS} & \text{MP} & \text{MD} & \text{CD} & \text{CS} & \text{TM} & \text{RK} \end{array} \\ \left[\begin{array}{ccccccccc} a_{77} & a_{78} & a_{79} & a_{76} & a_{75} & a_{74} & a_{73} & a_{72} & a_{71} \\ 0 & a_{88} & a_{89} & a_{86} & a_{85} & a_{84} & a_{83} & a_{82} & a_{81} \\ 0 & 0 & a_{99} & 0 & 0 & 0 & a_{93} & a_{92} & a_{91} \\ 0 & 0 & 0 & a_{66} & a_{65} & 0 & a_{63} & a_{62} & a_{61} \\ 0 & 0 & 0 & 0 & a_{55} & a_{54} & a_{53} & a_{52} & a_{51} \\ 0 & 0 & 0 & 0 & 0 & a_{44} & a_{43} & a_{42} & a_{41} \\ 0 & 0 & 0 & 0 & a_{35} & a_{34} & a_{33} & a_{32} & a_{31} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{22} & a_{21} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{11} \end{array}\right] \\ \begin{array}{ccccccccc} 8 & 7 & 6 & 6 & 4 & 4 & 2 & 1 & 0 \end{array} \end{array}.$$

Next, define the restriction matrices Q_j for $j = 1, \ldots, 9$ as

$$Q_1 = \left[\begin{array}{ccccccccc} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right],$$

$$Q_2 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, Q_3 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$Q_4 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, Q_5 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$Q_6 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, Q_7 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$Q_8 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, Q_9 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Finally, stack the products of the Q_j and A_0 matrices on top of $\left[I_j \; 0_{j\times(9-j)}\right]$ for $j =$

$1, \ldots, 9$. The results are

$$M_1 = \begin{bmatrix} 0 & a_{88} & a_{89} & a_{86} & a_{85} & a_{84} & a_{83} & a_{82} & a_{81} \\ 0 & 0 & a_{99} & 0 & 0 & 0 & a_{93} & a_{92} & a_{91} \\ 0 & 0 & 0 & a_{66} & a_{65} & 0 & a_{63} & a_{62} & a_{61} \\ 0 & 0 & 0 & 0 & a_{55} & a_{54} & a_{53} & a_{52} & a_{51} \\ 0 & 0 & 0 & 0 & 0 & a_{44} & a_{43} & a_{42} & a_{41} \\ 0 & 0 & 0 & 0 & a_{35} & a_{34} & a_{33} & a_{32} & a_{31} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{22} & a_{21} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{11} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$M_2 = \begin{bmatrix} 0 & 0 & a_{99} & 0 & 0 & 0 & a_{93} & a_{92} & a_{91} \\ 0 & 0 & 0 & a_{66} & a_{65} & 0 & a_{63} & a_{62} & a_{61} \\ 0 & 0 & 0 & 0 & a_{55} & a_{54} & a_{53} & a_{52} & a_{51} \\ 0 & 0 & 0 & 0 & 0 & a_{44} & a_{43} & a_{42} & a_{41} \\ 0 & 0 & 0 & 0 & a_{35} & a_{34} & a_{33} & a_{32} & a_{31} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{22} & a_{21} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{11} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, M_3 = \begin{bmatrix} 0 & 0 & 0 & a_{66} & a_{65} & 0 & a_{63} & a_{62} & a_{61} \\ 0 & 0 & 0 & 0 & a_{55} & a_{54} & a_{53} & a_{52} & a_{51} \\ 0 & 0 & 0 & 0 & 0 & a_{44} & a_{43} & a_{42} & a_{41} \\ 0 & 0 & 0 & 0 & a_{35} & a_{34} & a_{33} & a_{32} & a_{31} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{22} & a_{21} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{11} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$M_4 = \begin{bmatrix} 0 & 0 & a_{99} & 0 & 0 & 0 & a_{93} & a_{92} & a_{91} \\ 0 & 0 & 0 & 0 & a_{55} & a_{54} & a_{53} & a_{52} & a_{51} \\ 0 & 0 & 0 & 0 & 0 & a_{44} & a_{43} & a_{42} & a_{41} \\ 0 & 0 & 0 & 0 & a_{35} & a_{34} & a_{33} & a_{30} & a_{31} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{22} & a_{21} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{11} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, M_5 = \begin{bmatrix} 0 & 0 & a_{99} & 0 & 0 & 0 & a_{93} & a_{92} & a_{91} \\ 0 & 0 & 0 & 0 & 0 & a_{44} & a_{43} & a_{42} & a_{41} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{22} & a_{21} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{11} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$M_6 = \begin{bmatrix} 0 & 0 & a_{99} & 0 & 0 & 0 & a_{93} & a_{92} & a_{91} \\ 0 & 0 & 0 & a_{66} & a_{65} & 0 & a_{63} & a_{62} & a_{61} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{22} & a_{21} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{11} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}, M_7 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{22} & a_{21} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{11} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix},$$

$$M_8 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{11} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, M_9 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

It is straightforward to verify that the above M_j matrices are full rank for $j = 1, \ldots, 9$. Therefore, Rubio-Ramírez, Waggoner, and Zha (2010)'s sufficient rank condition is satisfied.

Since the necessary order and sufficient rank conditions are satisfied, the "Non-Recursive Identification: Money Supply Rule" model is also globally identified.

B.3. Verifying Global Identification of the Non-Recursive Identification: Interest Rate/Money Supply Rule Model

The third non-recursive identifcation scheme comes from Sims and Zha (2006). Under this identification scheme, the Fed switches between using the FFR and the MB as its policy instrument. Thus, I refer to this identification as the "Non-Recursive Identification: Interest Rate/Money Supply Rule" scheme (shown in Table B3).

Table B3: Non-Recursive Identifying Restrictions on the Impact Matrix A_0: Interest Rate/Money Supply Rule

	Behavioral Equation								
Shock / Variable	Risk Premium	Term Premium	Credit Supply	Credit Demand	Money Demand	Monetary Policy	Aggregate Supply	Aggregate Demand	Labor Supply
R_{Baa}	X								
R_{10yr}	X	X							
R_{CP}	X	X	X	X	X				
MI	X	X	X	X					
MB	X	X	X	X	X	X			
FFR	X	X	X		X	X			
RGDP	X	X	X	X	X		X	X	X
P	X	X	X	X	X			X	X
UR	X	X	X						X

Notes: Each column in this table represents a behavioral equation. The behavioral equations are labeled at the top by their respective structural shock. The row labels indicate which variables enter each behavioral equation at impact. An X entry denotes an unrestricted impact coefficient, while a blank space denotes a zero restriction.

Under this identification, the Fed no longer responds to RGDP and P prices at impact.

These short-run restrictions are mapped to the A_0 matrix

$$
A_0 = \begin{array}{c} \\ \mathrm{R_{Baa}} \\ \mathrm{R_{10yr}} \\ \mathrm{R_{CP}} \\ \mathrm{MI} \\ \mathrm{MB} \\ \mathrm{FFR} \\ \mathrm{RGDP} \\ \mathrm{P} \\ \mathrm{UR} \\ \\ q_j \end{array}
\begin{array}{c}
\begin{array}{ccccccccc} \mathrm{RK} & \mathrm{TM} & \mathrm{CS} & \mathrm{CD} & \mathrm{MD} & \mathrm{MP} & \mathrm{AS} & \mathrm{AD} & \mathrm{LS} \end{array} \\
\begin{bmatrix}
a_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
a_{21} & a_{22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & 0 & 0 & 0 & 0 \\
a_{41} & a_{42} & a_{43} & a_{44} & 0 & 0 & 0 & 0 & 0 \\
a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} & 0 & 0 & 0 \\
a_{61} & a_{62} & a_{63} & 0 & a_{65} & a_{66} & 0 & 0 & 0 \\
a_{71} & a_{72} & a_{73} & a_{74} & a_{75} & 0 & a_{77} & a_{78} & a_{79} \\
a_{81} & a_{82} & a_{83} & a_{84} & a_{85} & 0 & 0 & a_{88} & a_{89} \\
a_{91} & a_{92} & a_{93} & 0 & 0 & 0 & 0 & 0 & a_{99}
\end{bmatrix} \\
\begin{array}{ccccccccc} 0 & 1 & 2 & 4 & 4 & 7 & 8 & 7 & 6 \end{array}
\end{array} .
$$

Note the number of restrictions ($\sum_{j=1}^{9} q_j = 39$) is greater than $n(n-1)/2 = 36$. Therefore, the necessary order condition holds.

To verify Rubio-Ramírez, Waggoner, and Zha (2010)'s sufficient rank condition, rearrange the columns of A_0 from left to right in descending order so that

$$A_0 = \begin{array}{c} \\ \text{RGDP} \\ \text{P} \\ \text{FFR} \\ \text{UR} \\ \text{MB} \\ \text{MI} \\ \text{R}_{\text{CP}} \\ \text{R}_{\text{10yr}} \\ \text{R}_{\text{Baa}} \\ \\ q_j \end{array} \begin{array}{ccccccccc} \text{AS} & \text{AD} & \text{MP} & \text{LS} & \text{MD} & \text{CD} & \text{CS} & \text{TM} & \text{RK} \\ \left[a_{77} \right. & a_{78} & 0 & a_{79} & a_{75} & a_{74} & a_{73} & a_{72} & \left. a_{71} \right] \\ 0 & a_{88} & 0 & a_{89} & a_{85} & a_{84} & a_{83} & a_{82} & a_{81} \\ 0 & 0 & a_{66} & 0 & a_{65} & 0 & a_{63} & a_{62} & a_{61} \\ 0 & 0 & 0 & a_{99} & 0 & 0 & a_{93} & a_{92} & a_{91} \\ 0 & 0 & a_{56} & 0 & a_{55} & a_{54} & a_{53} & a_{52} & a_{51} \\ 0 & 0 & 0 & 0 & 0 & a_{44} & a_{43} & a_{42} & a_{41} \\ 0 & 0 & 0 & 0 & a_{35} & a_{34} & a_{33} & a_{32} & a_{31} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{22} & a_{21} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{11} \\ \\ 8 & 7 & 7 & 6 & 4 & 4 & 2 & 1 & 0 \end{array}.$$

Next, define the restriction matrices Q_j for $j = 1, \ldots, 9$ as

$$Q_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$Q_2 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, Q_3 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$Q_4 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, Q_5 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$Q_6 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, Q_7 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$Q_8 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, Q_9 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Finally, stack the products of the Q_j and A_0 matrices on top of $\begin{bmatrix} I_j \ 0_{j\times(9-j)} \end{bmatrix}$ for $j =$

$1, \ldots, 9$. The results are

$$M_1 = \begin{bmatrix} 0 & a_{88} & 0 & a_{89} & a_{85} & a_{84} & a_{83} & a_{82} & a_{81} \\ 0 & 0 & a_{66} & 0 & a_{65} & 0 & a_{63} & a_{62} & a_{61} \\ 0 & 0 & 0 & a_{99} & 0 & 0 & a_{93} & a_{92} & a_{91} \\ 0 & 0 & a_{56} & 0 & a_{55} & a_{54} & a_{53} & a_{52} & a_{51} \\ 0 & 0 & 0 & 0 & 0 & a_{44} & a_{43} & a_{42} & a_{41} \\ 0 & 0 & 0 & 0 & a_{35} & a_{34} & a_{33} & a_{32} & a_{31} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{22} & a_{21} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{11} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$M_2 = \begin{bmatrix} 0 & 0 & a_{66} & 0 & a_{65} & 0 & a_{63} & a_{62} & a_{61} \\ 0 & 0 & 0 & a_{99} & 0 & 0 & a_{93} & a_{92} & a_{91} \\ 0 & 0 & a_{56} & 0 & a_{55} & a_{54} & a_{53} & a_{52} & a_{51} \\ 0 & 0 & 0 & 0 & 0 & a_{44} & a_{43} & a_{42} & a_{41} \\ 0 & 0 & 0 & 0 & a_{35} & a_{34} & a_{33} & a_{32} & a_{31} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{22} & a_{21} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{11} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, M_3 = \begin{bmatrix} a_{77} & a_{78} & 0 & a_{79} & a_{75} & a_{74} & a_{73} & a_{72} & a_{71} \\ 0 & a_{88} & 0 & a_{89} & a_{85} & a_{84} & a_{83} & a_{82} & a_{81} \\ 0 & 0 & 0 & a_{99} & 0 & 0 & a_{93} & a_{92} & a_{91} \\ 0 & 0 & 0 & 0 & a_{35} & a_{34} & a_{33} & a_{32} & a_{31} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{22} & a_{21} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{11} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$M_4 = \begin{bmatrix} 0 & 0 & a_{66} & 0 & a_{65} & 0 & a_{63} & a_{62} & a_{61} \\ 0 & 0 & a_{56} & 0 & a_{55} & a_{54} & a_{53} & a_{52} & a_{51} \\ 0 & 0 & 0 & 0 & 0 & a_{44} & a_{43} & a_{42} & a_{41} \\ 0 & 0 & 0 & 0 & a_{35} & a_{34} & a_{33} & a_{32} & a_{31} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{22} & a_{21} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{11} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, M_5 = \begin{bmatrix} 0 & 0 & 0 & a_{99} & 0 & 0 & a_{93} & a_{92} & a_{91} \\ 0 & 0 & 0 & 0 & 0 & a_{44} & a_{43} & a_{42} & a_{41} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{22} & a_{21} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{11} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$M_6 = \begin{bmatrix} 0 & 0 & a_{66} & 0 & a_{65} & 0 & a_{63} & a_{62} & a_{61} \\ 0 & 0 & 0 & a_{99} & 0 & 0 & a_{93} & a_{92} & a_{91} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{22} & a_{21} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{11} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}, M_7 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{22} & a_{21} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{11} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix},$$

$$M_8 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{11} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, M_9 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

One can verify that the above M_j matrices are full rank for $j = 1, \ldots, 9$. Thus, Rubio-Ramírez, Waggoner, and Zha (2010)'s sufficient rank condition is satisfied.

Given the necessary order and sufficient rank conditions are satisfied, the "Non-Recursive Identification: Interest Rate/Money Supply Rule" model is also globally identified.

Appendix C.

Data Appendix

The MS-BVARs are estimated on a quarterly U.S. sample from 1960Q1 through 2018Q4. The data are gathered from the Federal Reserve Bank of St. Louis's FRED database and FRASER digital archive. Table C1 provides a detailed data description and corresponding sources. All time series are seasonally adjusted except for the interest rates and population. Quarterly observations are only available for real and nominal GDP. Thus, temporal aggregation is used to obtain quarterly observations from monthly data.

Table C1: Data Descriptions and Sources

Series	**Description**	**Source**	**Mnemonic**
RGDPQ	Real Gross Domestic Product, Billions of Chained 2012 Dollars	FRED	GDPC1
NGDPQ	Gross Domestic Product, Billions of Dollars	FRED	GDP
POP	Population, Thousands	FRED	B230RC0Q1735BEA
UR	Civilian Unemployment Rate	FRED	UNRATE
FFR	Effective Federal Funds Rate	FRED	FEDFUNDS
AMB	St. Louis Adjusted Monetary Base, Billions of Dollars	FRED	AMBSL
M2	M2 Money Stock, Billions of Dollars	FRED	M2SL
CURR	Currency Component of M1, Billions of Dollars	FRED	CURRSL
TVCKS	Traveler's Checks Outstanding, Billions of Dollars	FRED	TVCKSSL
$\mathrm{R}_{\mathrm{CP6M}}{}^{\dagger}$	Prime Commercial Paper Rate, 4-6 Month Maturity	FRASER	—
$\mathrm{R}_{\mathrm{CP3M}}$	3-Month Commercial Paper Rate	FRED	CP3M
$\mathrm{R}_{\mathrm{CPF3M}}$	3-Month AA Financial Commercial Paper Rate	FRED	CPF3M
$\mathrm{R}_{\mathrm{10yr}}$	10-Year Treasury Constant Maturity Rate	FRED	GS10
$\mathrm{R}_{\mathrm{Baa}}$	Moody's Seasoned Baa Corporate Bond Yield	FRED	BAA

† Weekly and monthly observations for the prime commercial paper rate are available from September 1929 to April 1997. The interested reader is referred to *G.13 Selected Interest Rates* at https://fraser.stlouisfed.org/title/1238.

C.1. Data Construction and Transformation

Table C2 defines the information set y_t. All variables in y_t are expressed in natural logarithms and scaled by 400 except for the interest rates and the unemployment rate, which are expressed in percents. The construction of MI and R_{CP} are discussed in detail below.

Table C2: Variables in the Information Set y_t

Variables	**Description**	**Construction**	**Transformation**
RGDP	Per Capita Real GDP	RGDPQ/POP	400 × Log-Level
P	Implicit GDP Price Deflator	100 × (NGDPQ/RGDPQ)	400 × Log-Level
UR	Unemployment Rate	UR	Percent
FFR	Federal Funds Rate	FFR	Percent
MB	Per Capita Monetary Base	AMB/POP	400 × Log-Level
MI	Per Capita Inside Money	See (C.1)	400 × Log-Level
$\mathrm{R_{CP}}$	Commercial Paper Rate	See (C.2)	Percent
$\mathrm{R_{10yr}}$	Ten-Year U.S. Treasury Bond Yield	$\mathrm{R_{10yr}}$	Percent
$\mathrm{R_{Baa}}$	Moody's Baa Corporate Bond Yield	$\mathrm{R_{Baa}}$	Percent

C.1.1. Per Capita Inside Money

An important distinction made in this paper is that between inside and outside money. Inside money is defined as the amount of bank deposits and other short-term liabilities issued by the banking system, whereas outside money consists of currency and reserves. In contrast to using the monetary base as a measure of outside money, there is not a directly observable measure of inside money. Therefore, I construct an inside money aggregate as follows.

The construction of the inside money aggregate relies on the readily available M1 and M2. However, it is important to note M1 and M2 contain inside *and* outside money components. For instance, M1 consists of currency (including vault cash), traveler's checks, demand deposits, and other checkable deposits (e.g., negotiable order of withdrawal accounts and credit union share draft accounts). M2 includes M1 plus saving deposits, small-denomination time deposits, and money market mutual funds.

With these definitions in mind, I first subtract the currency (CURR) and traveler's checks (TVCKS) components of M1 from M2. Next, I divide this aggregate measure of inside money by population (POP). Finally, I take the natural logarithm and multiply by 400 to acquire MI, such that

$$\mathrm{MI} = 400 \times \ln\left(\frac{\mathrm{M2} - \mathrm{CURR} - \mathrm{TVCKS}}{\mathrm{POP}}\right). \tag{C.1}$$

C.1.2. Commercial Paper Rate

Each time series in Table C1 has 236 available observations from 1960Q1 to 2018Q4 except for the three commercial paper rates. The three- and six-month commercial paper rate series ($\mathrm{R_{CP3M}}$ and $\mathrm{R_{CP6M}}$) are available from 1960Q1 to 1997Q2. Observations for the three-month financial commercial paper rate ($\mathrm{R_{CPF3M}}$) only date back to 1997Q1.

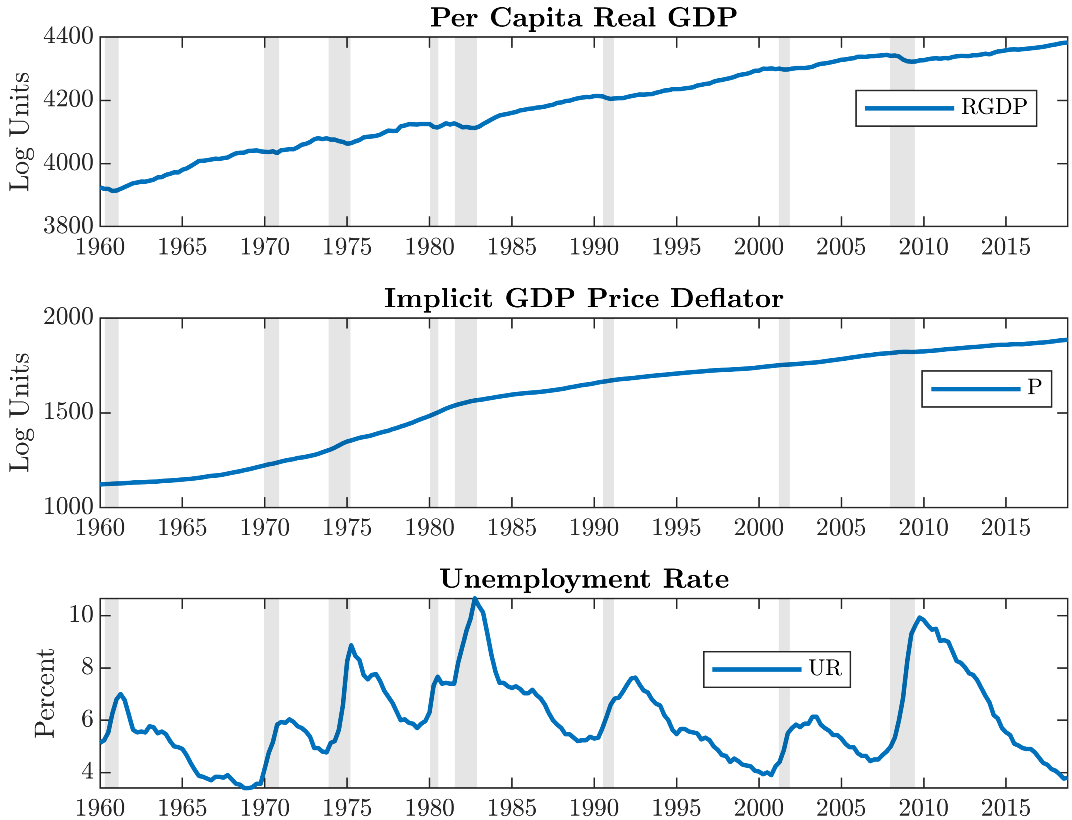
Following McCracken and Ng (2016), I splice the three commercial paper rates together to construct a new commercial paper rate series ($\mathrm{R_{CP}}$) that spans the entire sample period. The splicing procedure used to construct $\mathrm{R_{CP}}$ is

$$\mathrm{R_{CP}} = \begin{cases} \mathrm{R_{CP6M}} & \text{from 1960Q1 to 1971Q1,} \\ \mathrm{R_{CP3M}} & \text{from 1971Q2 to 1996Q4,} \\ \mathrm{R_{CPF3M}} & \text{from 1997Q1 to 2018Q4.} \end{cases} \tag{C.2}$$

Appendix D.

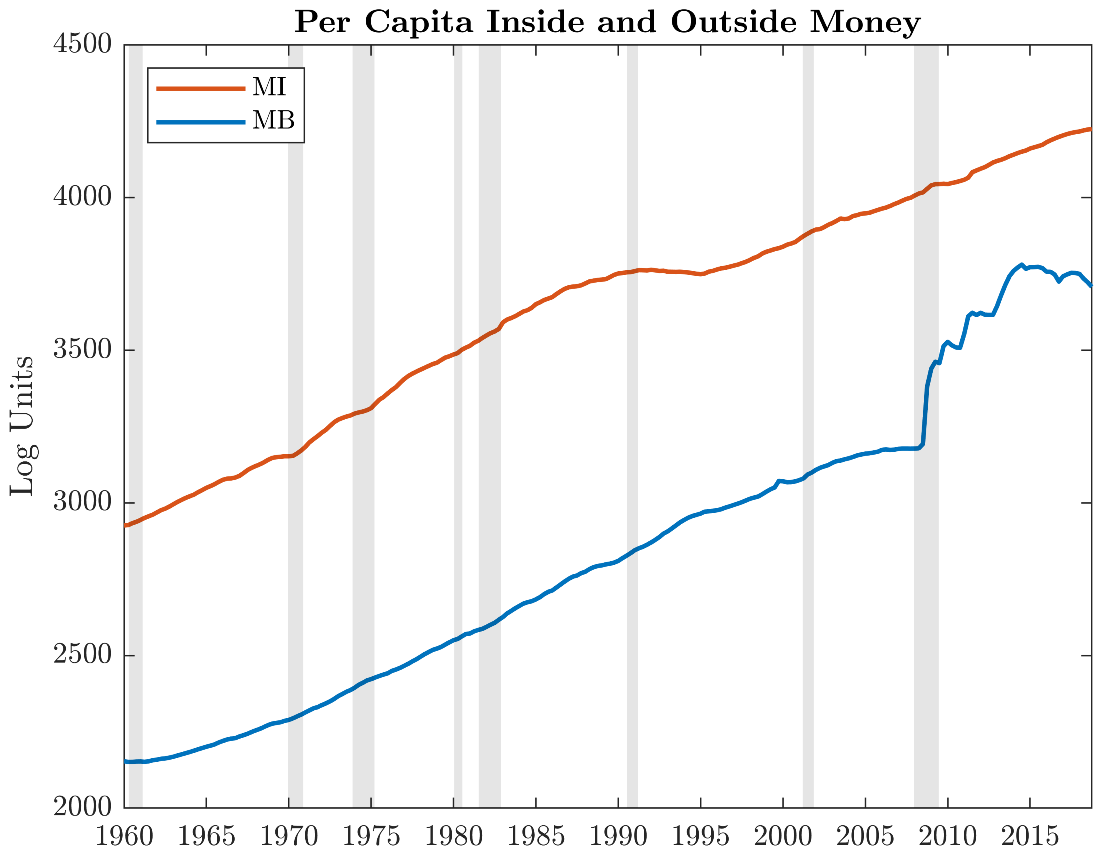
Time Series Plots

Figure D1: U.S. Macroeconomic Data, 1960Q1-2018Q4



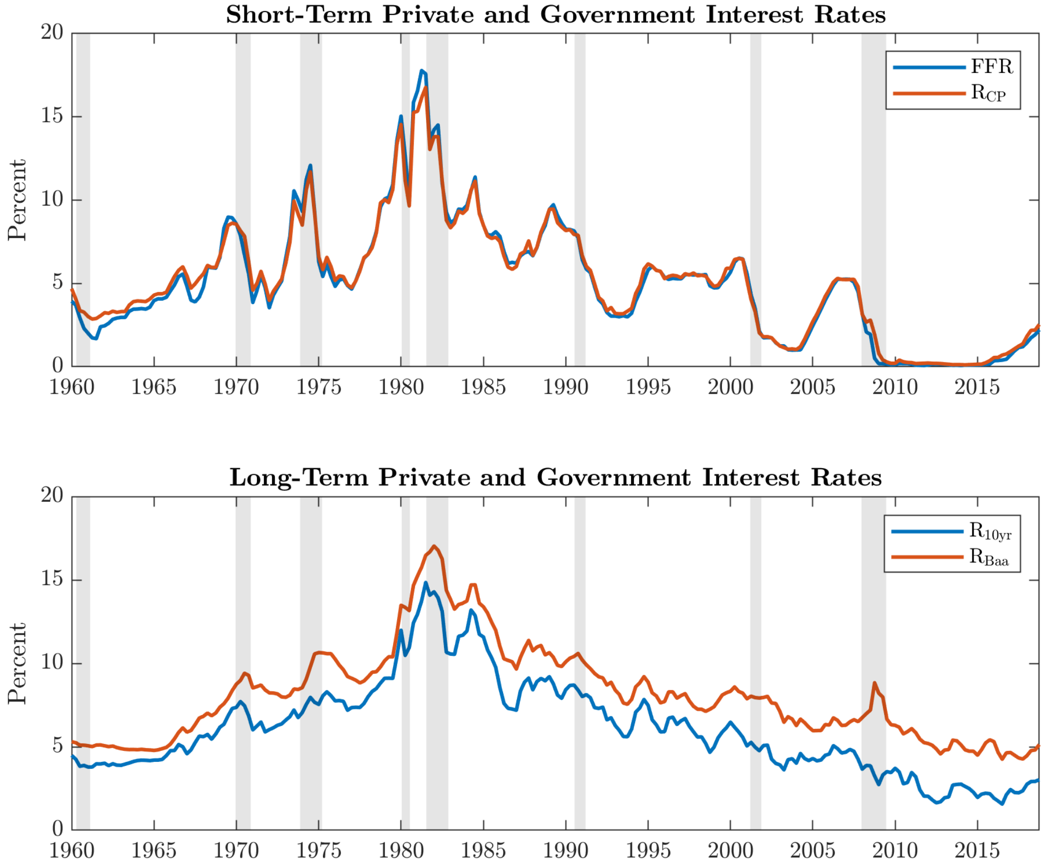
Notes: This figure plots U.S. per capita real GDP (RGDP), the implicit GDP price deflator (P), and the unemployment rate (UR) from 1960Q1 to 2018Q4. RGDP and P are in natural logarithms and scaled by 400. The shaded bars indicate NBER recession dates.

Figure D2: Per Capita Inside and Outside Money, 1960Q1-2018Q4



Notes: This figure plots per capita inside money (MI) and per capita outside money (MB) from 1960Q1 to 2018Q4. MI equals minus currency and traveler's checks. MB includes currency, vault cash, and reserves held by the banking system. MI and MB are in natural logarithms and scaled by 400. The shaded bars indicate NBER recession dates.

Figure D3: Short- and Long-Term Private and Government Interest Rates, 1960Q1-2018Q4



Notes: This figure plots U.S. short- and long-term private and government interest rates from 1960Q1 to 2018Q4. The top panel reports the effective fed funds rate (FFR) and the commercial paper rate (R_{CP}). The bottom panel shows the constant maturity yield on ten-year U.S. Treasury bonds (R_{10yr}) and the Moody's Baa corporate bond yield (R_{Baa}). The shaded bars indicate NBER recession dates.

Appendix E.

Computing Generalized Impulse Response Functions

This appendix overviews how to compute generalized impulse response functions (GIRFs) of an MS-BVAR. Computation of the GIRFs relies on an algorithm described in Karamé (2015) and Bianchi (2016). The GIRFs in the paper are constructed under the assumption that the regime is known with certainty at impact. However, these conditional GIRFs take into account the possibility of switching to future regimes. For the sake of brevity, the interested reader is referred to Karamé (2015) and Bianchi (2016) for more technical details.

E.1. Computing a GIRF

Sims and Zha (2006) and Sims, Waggoner, and Zha (2008) write the structural MS-BVAR

$$y_t' A_0(s_t) = x_t' A_+(s_t) + \varepsilon_t' \Xi^{-1}(s_t), \quad \varepsilon_t \sim \mathcal{N}(0_{n\times 1}, I_n), \quad t = 1, \ldots, T, \tag{E.1}$$

where all the parameters of each matrix can be gathered together to form $A_0 \equiv \{A_0(h)\}$, $A_+ \equiv \{A_+(h)\}$, and $\Xi \equiv \{\Xi(h)\}$ for $h = 1, \ldots, H$, where H denotes the total number of states or regimes. Let $\theta \equiv \{A_0, A_+, \Xi\}$ be the set of unknown impact and lag coefficients and SV factor loadings of the MS-BVAR. These parameters are estimated using the Metropolis-within-Gibbs MCMC sampler of Sims, Waggoner, and Zha (2008).

Suppose the first regime and the transition matrix Q are known at impact. It follows that one can predict a particular sequence of future regimes up to the $t+k$ horizon by computing

$$\text{Prob}\left[s_{t+k}, s_{t+k-1}, \ldots, s_{t+1} | s_t; \theta\right] = q_{s_{t+k-1}, s_{t+k}} \times \text{Prob}\left[s_{t+k-1}, \ldots, s_{t+1} | s_t; \theta\right]. \tag{E.2}$$

Conditional on each possible shock sequence ε_t and possible path of s_t, the optimal forecast for y_{t+k} can be calculated as

$$\mathrm{E}\left[y_{t+k}|s_{t+k},\ldots,s_t,\varepsilon_t;\theta\right] = c(s_t) + \sum_{j=1}^{p} \mathrm{E}\left[y_{t+k-j}|s_{t+k-1},\ldots,s_t,\varepsilon_t;\theta\right] A_j(s_t). \tag{E.3}$$

The shocked trajectory is the weighted average of all possible responses

$$\mathrm{E}\left[y_{t+k}|s_{t+k},\varepsilon_t;\theta\right] = \sum_{s_{t+k}} \cdots \sum_{s_{t+1}} \mathrm{E}\left[y_{t+k}|s_{t+k},\ldots,s_t,\varepsilon_t;\theta\right] \times \mathrm{Prob}\left[s_{t+k},\ldots,s_{t+1}|s_t;\theta\right]. \tag{E.4}$$

The conditional GIRF, given the initial regime s_t and shock ε_t, is defined as the difference between the shocked trajectory and a baseline (non-shocked) trajectory such that

$$GIRF(k,s_t,\varepsilon_t;\theta) = \mathrm{E}\left[y_{t+k}|s_{t+k},\varepsilon_t;\theta\right] - \mathrm{E}\left[y_{t+k}|s_{t+k},0;\theta\right]. \tag{E.5}$$

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